

Fractional order controller for satellite attitude stabilization based on magnetorquer

Nabil Mrani

Polydisciplinary Faculty of Errachidia - Moulay Ismail University - Morocco

Corresponding Author's Email: nmrani@gmail.com

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ABSTRACT

The paper presents a new approach for the three-axis attitude stabilization of a satellite in low orbit using magnetorquers. A comparative study is established between classical optimal control and a non-integer order control showing performance and robustness of the proposed method. The purpose of this paper is to develop a control law which is efficient, robust, easy to implement on the SCA, satisfying given specifications and maintaining stability and performance requirements, even in the presence of uncertainty about the intrinsic parameters of the satellite and strong external disturbances.

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Introduction:

The idea of fractional differentiation goes back four centuries, but the use of this tool in physics and particularly in the theory of automatic control dates from the second half of the last century [1],[2],[3]. However, the idea of applying the fractional control (non-integer order) in satellite attitude stabilization is originally introduced by the author in [4].

In other hand, the attitude stability of a satellite is of considerable importance for successful completion of a space mission. Thus, the essential function of an attitude control system of a satellite pointing earth is to maintain his attitude even in the presence of different sources of environmental disturbances such as torques due to gravity gradient, aerodynamic and solar forces... Etc, which may affect satellite attitude motion.

Different control approaches has been developed through the years and introduced in attitude control [5],[6],[7]. Different sensors and actuators has been also used, especially those of magnetic nature, because it provide a simple solution to the problem of generating a control torque on board a satellite. The use of such a tools for control purposes has been the subjects of many works since the early years of satellite missions [8],[9]. This paper deals with a method of LEO-satellite attitude stabilization using Magnetorquers and based on

fractional controllers, implementing control laws of non-integer order. The paper is organized as follows: in Section 2, a description of the dynamics of spacecraft is presented. In Section 3, a fractional control law for satellite attitude stabilization is proposed and discussed, while in Section 4 the results obtained with fractional controller in a simulation study are shown. the purpose of this section is to compare the behaviors of two types of controllers : the LQR one and a fractional controller represented in state space.

Satellite attitude dynamic model:

The satellite attitude dynamic model is given by the following equations system [13]:

$$\begin{aligned} I_R \ddot{\phi} + (I_R + I_L + I_T) \omega_0 \dot{\psi} + 4(I_T - I_L) \omega_0^2 \phi &= M_{cx} + M_{px} \\ I_T \ddot{\theta} + 3(I_R - I_L) \omega_0^2 \theta &= M_{cy} + M_{py} \\ I_L \ddot{\psi} - (I_R + I_L - I_T) \omega_0 \dot{\phi} + (I_T - I_R) \omega_0^2 \psi &= M_{cz} + M_{pz} \end{aligned} \quad (1)$$

Where:

- I_R, I_T et I_L the moments of inertia corresponding respectively to roll, pitch and yaw axis.
- ϕ, θ et ψ are roll, pitch and yaw assumed small in the attitude control.
- ω_0 : orbital angular rate.

$M_c \begin{bmatrix} M_{cx} \\ M_{cy} \\ M_{cz} \end{bmatrix}$ is the control torque

$M_p \begin{bmatrix} M_{px} \\ M_{py} \\ M_{pz} \end{bmatrix}$ is the disturbance torque.

In canonical form, the model (1) can be written in state space:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad (2)$$

where :

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 4\frac{I_L - I_T}{I_R} \omega_0^2 & 0 & 0 & 0 & 0 & \frac{I_R + I_T - I_L}{I_R} \omega_0^2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3\frac{I_L - I_R}{I_T} \omega_0^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{I_R + I_L - I_T}{I_T} \omega_0 & 0 & 0 & \frac{I_R - I_T}{I_L} \omega_0^2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{I_R} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{I_T} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_L} \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix} \text{ and } u = \begin{bmatrix} M_{cx} + M_{px} \\ M_{cy} + M_{py} \\ M_{cz} + M_{pz} \end{bmatrix}$$

Control laws:

Fractional systems are becoming increasingly in various research fields. However, despite the gradual interest one takes in these systems, engineering sciences applications are still in an embryonic stage. It is reported in recent works that controllers making use of fractional order derivatives and integrals gives performance and robustness results superior to those achieved with conventional controllers [10],[11],[12]. Several other methods are used in attitude stabilization such as PID control, LQR, pole placement, H2, H ∞ , etc [14],[15]. In this section, in order to show the interest of

the method proposed in this work, we will compare fractional control and LQR methods.

A. Linear quadratic control

Consider the controlled process described by the state equation.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (3)$$

The goal is to find a solution $u(t)$ that minimizes the continuous-time cost functional.

$$J = \frac{1}{2} \int_{t_0}^T (x^T(t)Qx(t) + u^T(t)Ru(t)) dt \quad (4)$$

Where the matrices R and Q are positive definite.

Under some controllability conditions, solution of such problem is given by:

$$u(t) = -Kx(t)$$

where:

$$K = R^{-1}B^T P \quad (5)$$

and P is solution of the well-known Riccati equation:

$$A^T P + AP - A^T PBR^{-1}B^T P + Q = 0$$

B. Non integer control

Consider the satellite attitude dynamics modeled by the system (2). The control law proposed here is of fractional type and specially α -order derivative of the state $x(t)$, given as:

$$u(t) = -Kx^{(\alpha)}(t) \quad (6)$$

where α is a real non integer arbitrary number and which denotes the order of derivation.

The linear fractional system will become:

$$\dot{x}(t) = Ax(t) - BKx^{(\alpha)}(t); \quad x(0) = x_0.$$

Note that the fractional derivative of a function is defined by grünwald-Letnikov [16] as:

$$x(t)^\alpha = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{k=0}^{\infty} (-1)^k \binom{\alpha}{k} x(t - kh)$$

where :
$$\binom{\alpha}{k} = \frac{\Gamma(\alpha + 1)}{\Gamma(k + 1)\Gamma(\alpha - k + 1)}$$

and where Γ refers to the Gamma function which is the extension of the factorial function in the non-integer case.

Simulation results:

Consider the simulation parameters below:

$$\omega_0 = 0.0011 \frac{rad}{s}; \quad I_R = 90 \text{ m}^2\text{kg}$$

$$I_T = 120 \text{ m}^2\text{kg}; \quad I_L = 50 \text{ m}^2\text{kg}$$

and initial conditions :

$$x(0) = (1^\circ, 0.1^\circ/s, 1^\circ, 0.1^\circ/s, 1^\circ, 0.1^\circ/s)^T$$

The matrix K calculated from (5) is given by:

$$K = \begin{bmatrix} 0.9996 & 13.4513 & 0.0000 & -0.0000 & 0.0019 & 0.0000 \\ -0.0000 & -0.0000 & 0.9998 & 15.5230 & -0.0000 & -0.0000 \\ -0.0019 & 0.0000 & -0.0000 & -0.0000 & 1.0000 & 10.0497 \end{bmatrix}$$

It should be noted now that $\alpha = 0$ corresponds to an LQR control because the action control given in (6) becomes $u(t) = -Kx(t)$.

Figures (1a), (1b) and (1c) illustrate the evolution of the Roll, Pitch and Yaw angles versus time for different values of α between -0.5 and 0.5, within a step of 0.1.

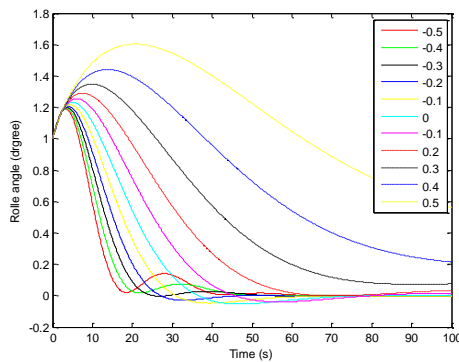


Fig. 1a: Evolution of the Roll angle versus time.

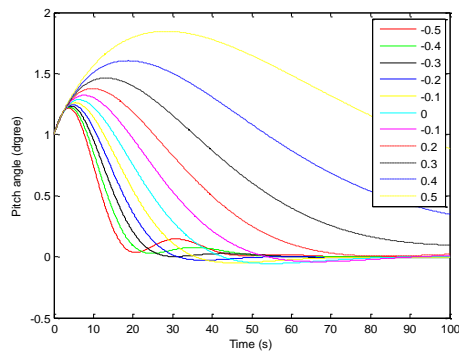


Fig. 1b: Evolution of the Pitch angle versus time.

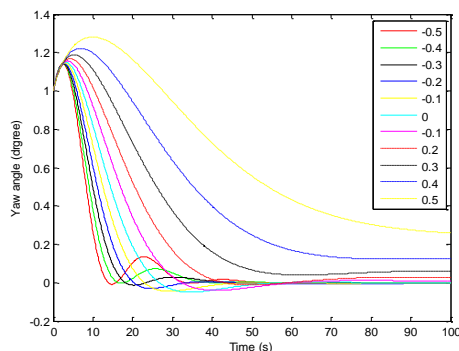


Fig. 1c: Evolution of the Yaw angle versus time.

It is noticed that the angles are settling down to zero for different α . However, the settling time for negative values of α is shorter than those for positive ones. It can

be seen that better precision and shorter settling time are obtained when α increases.

Figures (2a), (2b) and (2c) illustrate evolution of the Roll, Pitch and Yaw angular velocity versus time for different values of α between -0.5 and 0.5, within a step of 0.1.

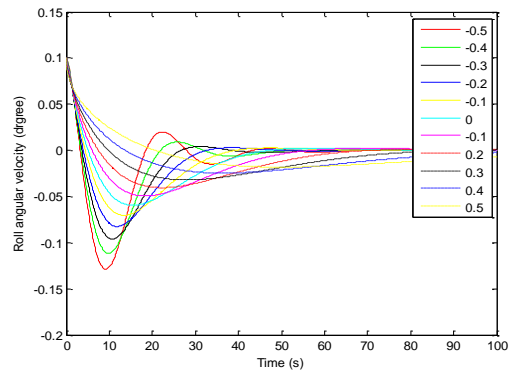


Fig. 2a: Evolution of Roll angular velocity versus time.

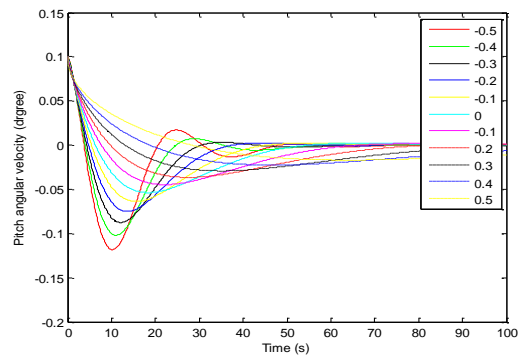


Fig. 2b: Evolution of Pitch angular velocity versus time.

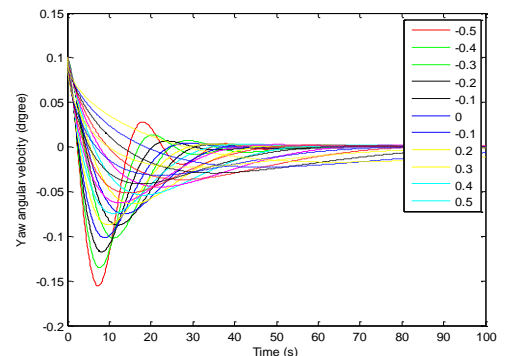


Fig. 2c: Evolution of Yaw angular versus time.

These curves present an overshoot increasingly important for negative values of α , and which tends rapidly to zero. To show the contribution of the fractional approach in terms of rapidity, we propose to study the system settling time, i.e. the angle and angular velocity time response for roll, pitch and yaw.

The following table 1 and table 2 shows the results obtained:

Table: 1. Euler angels settling time versus α

α	Roll	Pitch	Yaw
-0.5	36.1	40.5	28.8
-0.4	37.9	42.6	30.1
-0.3	21.3	24.6	16
-0.2	24.3	28.1	18.1
-0.1	28.1	45.9	20.7
0	50.8	60.5	24.1
0.1	41.1	48.9	28.9
0.2	54.6	66.3	36.7
0.3	100	100	100
0.4	100	100	100
0.5	100	100	100

Note: Units are in terms of s

Table: 1. Euler angels velocity settling time versus α

α	Roll	Pitch	Yaw
-0.5	40.2	44.3	42.8
-0.4	42.5	46.3	35.2
-0.3	25	25.3	26.5
-0.2	29.7	33.9	22.7
-0.1	35.1	40.2	26.5
0	41.3	47.7	30.9
0.1	48.9	57	36
0.2	58.5	68.8	42.2
0.3	71.1	84.5	50.2
0.4	87.9	100	60.6
0.5	100	100	74.3

Note: Units are in terms of s

The above two tables show clearly the growing trend of the settling time for positive values of α and which both present an optimum for $\alpha=-0.3$.

Figures (3a), (3b) and (3c) illustrate graphically evolution of settling time of the Roll, Pitch and Yaw angles and angular velocity versus time for α between -0.5 and 0.5, within a step of 0.1.

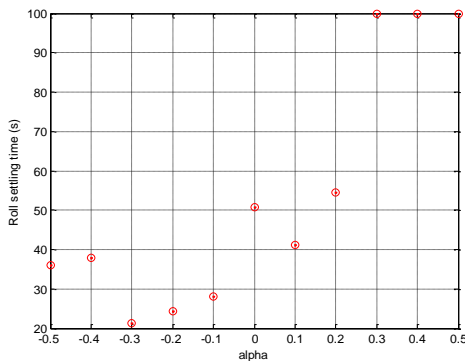


Fig. 3a: Evolution of Roll settling time versus α .

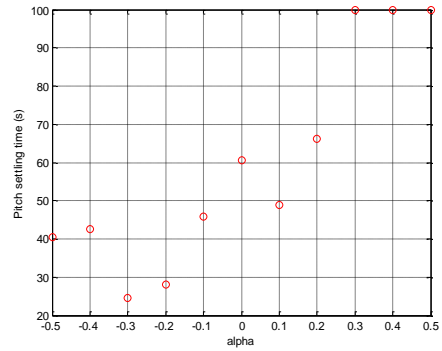


Fig. 3b: Evolution of Pitch settling time versus α .

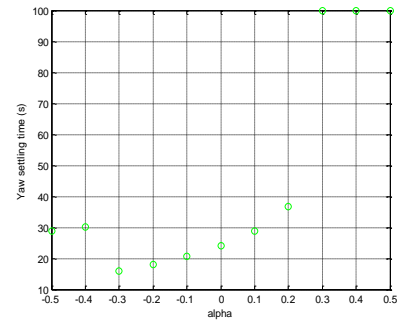


Fig. 3c: Evolution of Yaw settling time versus α .

It should be noted that the accuracy also increases with the order of fractional differentiation α , as that is shown below in figure.4a and figure.4b, for Euler's angles and angular velocity for the same values range of the fractional order α .

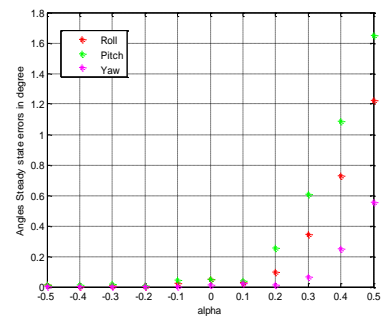


Fig. 4a: Angles accuracy versus α .

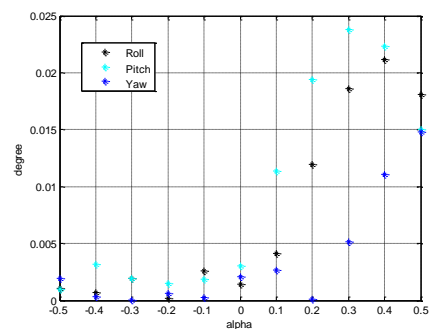


Fig. 4b: Angular velocity steady state errors.

Table3 summarize the steady state error taken at 150 second, for roll, pitch and yaw angles with respect to different values of $\alpha \in [-0.5, 0.5]$. It can be seen that roll angle steady state error, for example, passes from 0.0154, for $\alpha = -0.5$, to 1.2257 for $\alpha = 0.5$.

Table: 3. Angles steady state error versus α

α	Roll	Pitch	Yaw
-0.5	0.0154	0.0107	0.0026
-0.4	0.0026	0.0146	0.0028
-0.3	0.0071	0.0186	0.0028
-0.2	0.0013	0.0064	0.002
-0.1	0.026	0.0454	0.0013
0	0.0512	0.0493	0.0109
0.1	0.027	0.0374	0.0242
0.2	0.0953	0.253	0.0123
0.3	0.3462	0.608	0.0663
0.4	0.7299	1.0852	0.2493
0.5	1.2257	1.6488	0.5568

Note: Units are in terms of s

Roll, pitch and yaw angles velocity steady state error are given in table 4, where we can clearly see the growing trend of the SSE for positive values of α

Table: 4. Angles velocity steady state error versus α

α	Roll	Pitch	Yaw
-0.5	0.0011	0.001	0.0019
-0.4	0.0007	0.0032	0.0003
-0.3	0.0019	0.002	0
-0.2	0.0002	0.0015	0.0006
-0.1	0.0026	0.0019	0.0002
0	0.0014	0.003	0.0021
0.1	0.0041	0.0114	0.0027
0.2	0.0119	0.0194	0.0001
0.3	0.0186	0.0238	0.0052
0.4	0.0211	0.0224	0.0111
0.5	0.0181	0.0151	0.0148

Note: Units are in terms of s

Note that the good accuracies obtained for the negative values, is due to the fact that for those values we introduce a fractional integrator which insure good performances but to the detriment of other criteria, especially the control energy, such as it will be shown after.

Both accuracy and settling time increase with α . This justifies the idea of trying to negotiate other criteria for an optimal choice of the order α for algorithmic implementation. Thus, we propose to study the energy of the control action provided to the magnetorquers, control torque evolution versus time and control torque overshoot with respect to α . Figure 5 shows how the energy decreases with α , and Figures 6a, 6b and 6c

show how the control torque varies versus time for each Euler angles and for different α values.

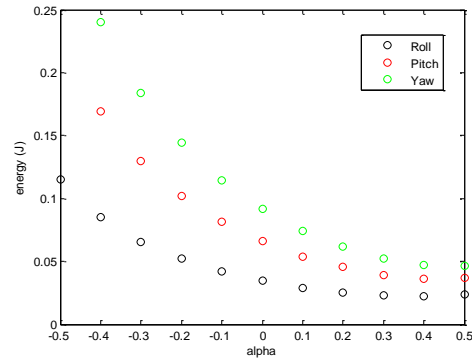


Fig. 5: Control-action energy of the Euler angles.

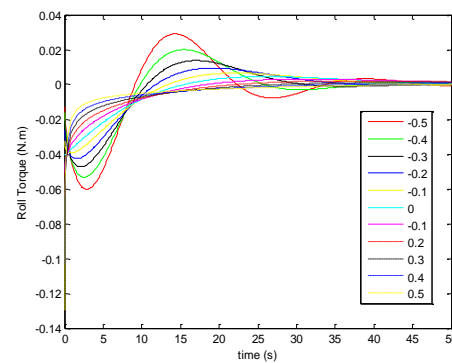


Fig. 6a: Control torque applied to the roll versus α .

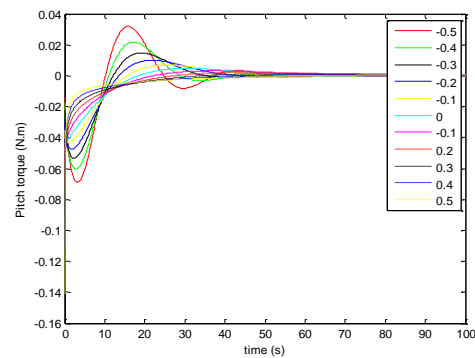


Fig. 6b: Control torque applied to the pitch versus α .

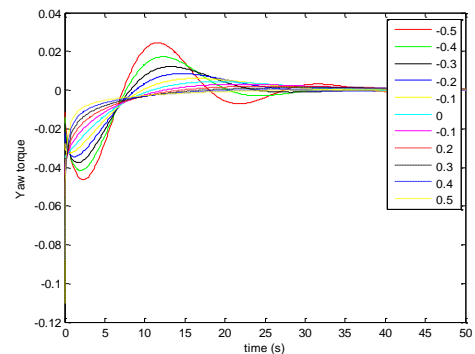


Fig. 6c: Control torque applied to the yaw versus α .

We can see that solicitation in control torque is less important for a control law with fractional derivation rather than control law with fractional integration. Figure 7 shows the evolution of the control action overshoot according to α values between -0.5 and 0.5.

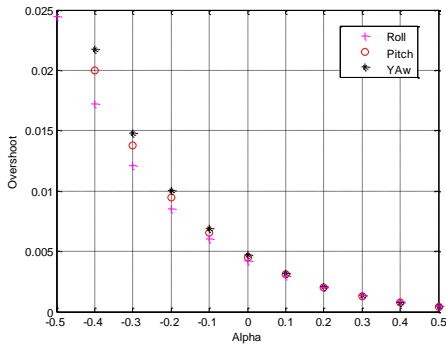


Fig. 7: Evolution of the overshoot versus α .

We clearly see the decay of the first overshoot when the fractional order increases. Thus, solicitation on control torque is less important for a control law with fractional derivation rather than a fractional integration.

Table 5 explicitly summarizes and recapitulates the numerical values of overshoots relating to roll, pitch and yaw angles for $\alpha \in [-0.5, 0.5]$.

Table 5. Overshoot for different values of α .

α	Roll	Pitch	Yaw
-0.5	0.0290	0.0319	0.0244
-0.4	0.0200	0.0148	0.0172
-0.3	0.0138	0.0148	0.0121
-0.2	0.0095	0.0101	0.0085
-0.1	0.0065	0.0069	0.0060
0	0.0045	0.0047	0.0042
0.1	0.0031	0.0032	0.0029
0.2	0.0020	0.0021	0.0020
0.3	0.0013	0.0012	0.0013
0.4	0.00068	0.00023	0.00081
0.5	0.00012	0.00061	0.00045

Note: Units are in terms of $N.m$

Note that the good accuracies obtained for the negative values, is due to the fact that those values introduce a fractional integration which insure good performances but it requires more energy. $\alpha = -0.3$, seems to be optimal for controller performances.

Conclusion:

The idea of the fractional calculus application to control theory was introduced in many works, and its advantages were proved. However, the realization of fractional-order controllers for satellite attitude stabilization based on magnetic torque has not been

investigated. In this paper, Earth pointing satellite attitude stabilization with fractional control law method is proposed and examined to stabilize the satellite attitude movement. It can be seen that fractional-order controllers are more flexible, robust, more accurate and faster in term of time response than LQR control for some specific values of α .

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